Observation of a coherent backscattering effect with a dipolar source for elastic waves: Highlight of the role played by the source

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We report an experimental evidence of the role played by the source on the coherent backscattering effect (CBE) for elastic waves. The experiment is carried out in a chaotic cavity consisting of a silicon plate whose shape is a quarter stadium. With a monopolar source, it has already been shown that the time-integrated squared amplitude at the point source is twice as large as at the other points around the source. Here, by using a dipolar source, we show that we instead obtain two peaks with the same axis as the dipole one's. The shape of this ''bicone'' is well explained with a modal theory assuming that the source may be modeled by two sources with opposite phases. Then, the theory is generalized to any multipolar emitter and/or receiver. Particularly, we study CBE when emitter and receiver are reciprocal.

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I. INTRODUCTION

Contrary to our first intuition, a wave that undergoes multiple scattering by random heterogeneities or multiply reflected by complex boundaries keeps some coherence. One of the consequences of that is the well-known coherent backscattering effect (CBE) recorded first for multiple-scattering media in optics [1] and later in acoustics [2]. When a multiple-scattering media is illuminated by a monochromatic plane wave, the interference between a path and its reciprocal counterpart implies that on average the reflected intensity is higher in the backscattering direction than in the other directions. In the far field, i.e., when the source is far from the multiple-scattering media, this effect gives a spatial dependence of the backscattered intensity looking like a ''cone.'' More recently, experimental evidence of the near field coherent backscattering effect for transient elastic waves propagating in a chaotic cavity has been reported [3]. In this experiment, the elastic energy is injected directly inside the medium by an isotropic pointlike source. In this case, the CBE looks like an axisymmetric bump around the source position. It is important to mention that working with pulsed waves avoids averaging to make this effect emerge: it is clearly observable on a single realization. Ensemble averaging is replaced by averaging on the independent frequencies included in the pulse. Even more recently, Weaver and Lobkis [4] obtained the experimental confirmation that for reverberation times larger than a characteristic value, the so-called Heisenberg time, the enhancement value is not two as it is the case for multiple-scattering media but three [5]. This comes as a consequence of the statistics of the eigenfunctions inside a chaotic cavity. But up to now, the role played by the source on the effect has not been studied except very recently in a theoretical paper by Tiggelen *et al.* [6]. What happens for multipolar (non-isotropic) pointlike emitter and receiver? Do we find the same ''universal'' shape with the same enhancement as in the monopolar case? If not, what is the shape and can we deduce it from the monopolar shape? These questions are relevant for at least two reasons. First, from a fundamental point of view, it arises explicitly the role of the emission and reception on the CBE shape. Second, from a more practical point of view, in seismology, the recording of the CBE could bring the evidence that there are multiple-scattering process in the earth [7]. But in the case of earthquakes, the sources are multipolar by nature. Their influences on the CBE have thus to be understood.

In this paper, we present experimental results showing that if a pointlike dipolar source is used, two peaks instead of one are observed that produce a ''bicone.'' Then we show that its shape is well explained within a modal theory in which the source is modeled by two sources in opposite phases. This experiment highlights the case when the emission is dipolar and the reception monopolar. Finally, we give a general expression for any kind of emitter and receiver. Especially, we focus on the case where emitter and receiver are reciprocal: for instance, when similar devices generate and record the field.

II. EXPERIMENT

The experimental setup is as follows. We use a chaotic cavity consisting of a silicon plate whose shape is a quarter stadium. Its area is 2335 mm^2 while its thickness is 0.5 mm. The sources are transducers coupled to aluminum cones. Their tip sizes are much less than the characteristic wavelength of the elastic waves in the plate (2.5 mm). Therefore, the source can be considered pointlike. A monopolar source is obtained by using a longitudinal transducer and a dipolar source by using a transversal one. The cone angle is chosen to minimize the pulse dispersion between the base and the tip [8]. The bandwidth lies between 300 kHz and 1.5 MHz. The normal displacements are measured by a heterodyne interferometer whose optical beam is focused through a lens on a spot 100 μm sized. The advantage of using such an interferometer lies in the fact that it gives an absolute measurement without perturbing the propagation of the elastic waves. At time $t=0$, a short pulse (a few microseconds long) is transmitted in the plate at point $\mathbf{r}_0$. After 200 μs, the elastic

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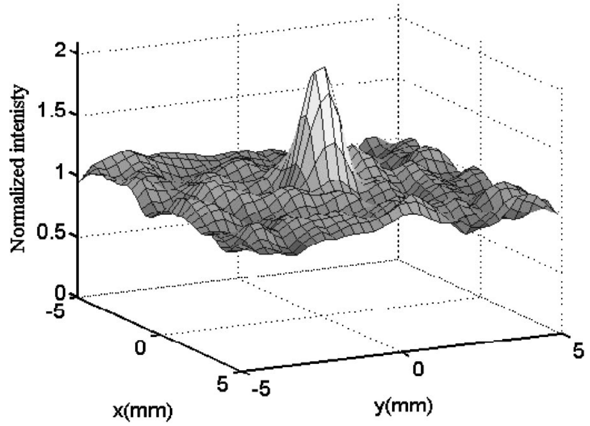

FIG. 1. Intensity pattern of the field for a monopolar source.

field is uniformly distributed all over the cavity. The characteristic decay time over which the squared amplitude is decreased by a factor $1/e$ is equal to 1.1 ms. To determine the spatial distribution of the stationary intensity, we integrate the square of the amplitude between time $T_1 = 200$ μs to avoid the first reflections and $T_2 = 5$ ms, which is imposed by the attenuation

$$I(\mathbf{r},\mathbf{r}_0) = \int_{T_1}^{T_2} \Psi^2(\mathbf{r},\mathbf{r}_0;t)\,dt. \tag{1}$$

We present in Fig. 1 the spatial intensity distribution previously found for a monopolar source [3]. If the same measurement is repeated with a dipolar source, the shape is totally different (Fig. 2). We do not obtain a single peak at the source location, as is the case for a monopolar source, but two peaks producing a ‘‘bicone’’ with the same axis than the dipolar source’s. The distance between the two peaks is about 2 mm, which is roughly half a wavelength.

III. THEORY

Now, we show that this bicone may be explained by invoking the superposition principle. The dipolar source is the superposition of two monopolar sources with opposite phases and separated by a distance vector $\mathbf{d}$. A pointlike dipole is obtained when $\|\mathbf{d}\|$ approaches 0. Linearity of the system enables us to write the dipolar field Ψ_d using the monopolar solutions Ψ_m

$$\Psi_d(\mathbf{r},\mathbf{r}_0,t) = \Psi_m\left(\mathbf{r},\mathbf{r}_0 - \frac{\mathbf{d}}{2},t\right) - \Psi_m\left(\mathbf{r},\mathbf{r}_0 + \frac{\mathbf{d}}{2},t\right). \tag{2}$$

In order to simplify the notations, the center of the dipole is taken as the origin of the referential $(\mathbf{r}_0 = \mathbf{0})$. Taking the ensemble average of the squared amplitude gives us

$$\langle \Psi_d^2(\mathbf{r},\mathbf{r}_0,t)\rangle = \left\langle \Psi_m^2\left(\mathbf{r},-\frac{\mathbf{d}}{2},t\right)\right\rangle + \left\langle \Psi_m^2\left(\mathbf{r},+\frac{\mathbf{d}}{2},t\right)\right\rangle - 2\left\langle \Psi_m\left(\mathbf{r},+\frac{\mathbf{d}}{2},t\right)\Psi_m\left(\mathbf{r},-\frac{\mathbf{d}}{2},t\right)\right\rangle. \tag{3}$$

The computation of each term may be done using the modal decomposition of the monopolar response field

$$\Psi_m(\mathbf{r},\mathbf{r}_0,t) = \sum_{n=0}^{\infty} F(\omega_n)\frac{\sin(\omega_n t)}{\omega_n}\Phi_n(\mathbf{r_0})\Phi_n(\mathbf{r}), \tag{4}$$

where $F(\omega_n)$ is the Fourier transform of the excitation function $f(t)$ and $\Phi_n(\mathbf{r})$ is the eigen-function related to the nth eigenpulsation ω_n. The excitation function is chosen as a sufficiently narrow band in order that $\Delta\omega/\omega_c \ll 1$, where $\Delta\omega$ is the width of $F(\omega_n)$ and ω_c is the central pulsation. Hence, statistical properties of ω_n and $\Phi_n(\mathbf{r})$ are considered constant inside the excitation band. Nevertheless, $\Delta\omega$ is sufficiently large in order to excite many modes inside the cavity. In other words, $\Delta\omega \gg \delta\omega$, where $\delta\omega$ is the mean spacing level between eigenpulsations.

All three terms of Eq. (4) may be extracted from the expression, $\langle \Psi_m(\mathbf{r},\mathbf{r}_A,t)\Psi_m(\mathbf{r},\mathbf{r}_B,t)\rangle$, where $\mathbf{r}$, $\mathbf{r}_A$, and $\mathbf{r}_B$ are any of the positions of three points inside the cavity. The way to compute this quantity is similar to that done in [3,4]. These three points must be apart more than a few wavelengths from the edge, otherwise, the boundary conditions perturb the statistical properties of the eigenfunctions. Finally, defining the spatial autocorrelation function of the modes, i.e., $\langle \Phi(\mathbf{r})\Phi(0)\rangle/\langle \Phi^2\rangle$ as $L(\mathbf{r})$ and the fourier transform of the autocorrelation of the modal density as $b(T)$, one obtains

$$I_d(\mathbf{r},\mathbf{r}_0 = 0;T) = 1 + 2[2 - b(T)]\frac{[L(\mathbf{r}+d/2) - L(\mathbf{r}-d/2)]^2}{L(0) - L(\mathbf{d})}. \tag{5}$$

T is the center of the time integration window. The analytic expression of the function b can be found in Refs. [5,9]. It has been shown that the factor $[2 - b(T)]$ starts from one to reach two as T increases. The characteristic ‘‘break time’’ is the Heisenberg time of the cavity. This time is equal to the modal density, $n(\omega_c)$. For a two-dimensional (2D) problem, the spatial autocorrelation for chaotic cavities is a Bessel function of the first kind [10], i.e., $L(\mathbf{r}) = J_0(2\pi\|\mathbf{r}\|/\lambda_c)$. In

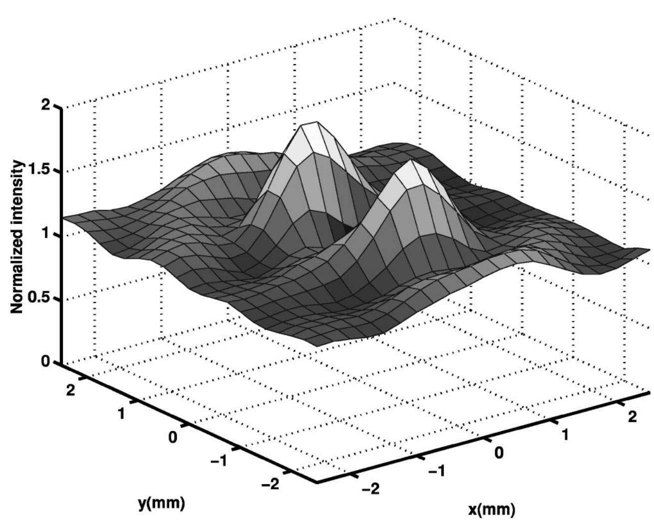

FIG. 2. Intensity pattern of the field for a dipolar source.

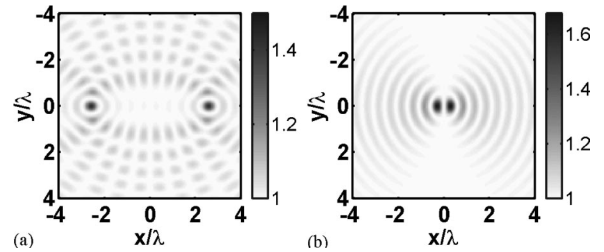

FIG. 3. Theoretical intensity pattern for two monopolar sources opposite in phase separated by a distance $\|\mathbf{d}\|$. (a) $\|\mathbf{d}\|=5\lambda_c$ and (b) $\|\mathbf{d}\|=\lambda_c/100$.

our experiments, we do not notice any significant temporal evolution with T. In the discussion section of this paper, we give some possible explanations for this fact. But, as we notice in Eq. (5), the spatial and temporal dependencies are factorized: thus, the choice of a time T modifies only the enhancement by a factor included between one and two. So it is justified to focus only on the spatial dependence of the Coherent Backscattering Effect. In the following, T is chosen much less than the Heisenberg time of the cavity. In Fig. 3, we have plotted theoretical spatial distributions of the averaged intensity for two distances between the two monopolar sources, respectively, $\|\mathbf{d}\|=5\lambda_c$ and $\|\mathbf{d}\|=\lambda_c/100$. When the sources are far away compared to the wavelength, the two peaks are separated by $\|\mathbf{d}\|$ [Fig. 3(a)]. On the other hand, when the sources are close [Fig. 3(b)], the distance between the peaks is much larger than $\|\mathbf{d}\|$ and seems to be roughly equal to half the wavelength.

The interpretation is the following: for $\|\mathbf{d}\|\gg\lambda_c$ the third term of Eq. (3), turns out to be zero so that background noise obtains two independent peaks. The enhancement factor is only 1.5 because the background noise related to one peak adds to the background noise of the other peak. If $\|\mathbf{d}\|$ becomes comparable to λ_c, the two peaks cannot be considered independent any longer. When the two sources are very close ($\|\mathbf{d}\|\ll\lambda_c$), we still observe two peaks. But now the peaks are separated by about $0.6\lambda_c$ and the maximum enhancement is equal to 1.68 independent of the short distance between the two sources. A Taylor expansion ($\|\mathbf{d}\|/\lambda_c\to 0$), of Eq. (5) is valid and leads to a simple analytical solution

$$I_d(\mathbf{r},\mathbf{r}_0=0;T)=1+2[2-b(T)]J_1^2(2\pi\|\mathbf{r}\|/\lambda_c)\cos(\theta). \tag{6}$$

θ is the angle between vectors $\mathbf{r}$ and $\mathbf{d}$. We have recorded the CBE around two central frequencies, 600 kHz [Fig. 4(a)] and 1.3 MHz [Fig. 4(c)]. For each frequency, the theoretical prediction [Eq. (6)] has been plotted [Figs. 4(b) and 4(d)]. The central wavelength λ_c comes from a direct measurement of the dispersion relation. Indeed, the interferometer records the dispersive flexural mode excited by the dipolar source [11]. The agreement between experiments and theory is excellent.

IV. DISCUSSION

This experiment with a multipolar source leads to the more general problem of the influence of emitter and receiver characteristics on the backscattering enhancement. Following our works, B. Van Tiggelen *et al.* [6] have extended the theory of multiple scattering for seismic elastic waves. Indeed, natural seismic sources are generally multipolar. Moreover, the receiver polarity must also be considered. This leads to figure out a more global understanding of the coherent backscattering enhancement, which cannot be separated from emission and reception process. We consider here a simple theory based on the modal decomposition of wave functions for quasipunctual multipolar sources. The field $\Psi_g(\mathbf{r},\mathbf{r_0};t)$ coming from a multipolar source and also recorded by a multipolar receiver is deduced from the monopolar field Ψ_m

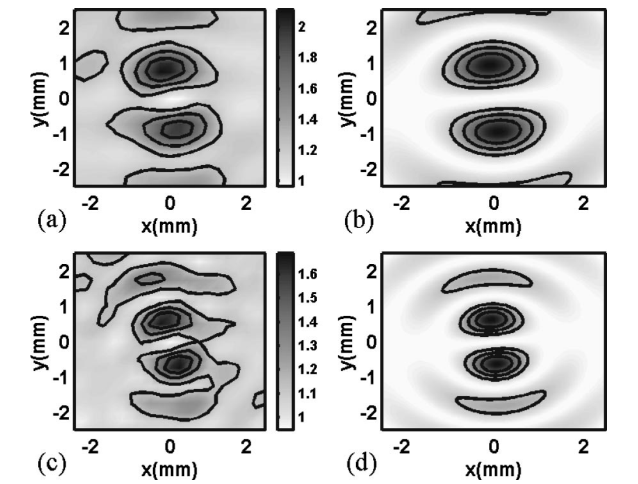

FIG. 4. Intensity pattern of the field for a dipolar source: (a) and (c): experimental results, respectively, for a 600 kHz central frequency pulse and a 1300 kHz pulse. (b) and (d): predictions for the corresponding wavelength, i.e., $\lambda_c=3.22$ and 2.15 mm.

$$\Psi_g(\mathbf{r},\mathbf{r}_0;t)=\hat{B}_{\mathbf{r}}\hat{A}_{\mathbf{r}_0}\Psi_m(\mathbf{r},\mathbf{r}_0;t), \tag{7}$$

where $\hat{A}_{\mathbf{r}_0}$ and $\hat{B}_{\mathbf{r}}$ are two operators that describe, respectively, the source (acting on $\mathbf{r}_0$ variables) and the receiver (acting on $\mathbf{r}$). For example, in our experimental situation, the source is dipolar, i.e., $\hat{A}_{\mathbf{r}_0}=\boldsymbol{\gamma}\nabla_{\mathbf{r}_0}$, where $\boldsymbol{\gamma}$ is a vector that represents the strength and the direction of the dipolar emission. The laser spot forms a monopolar receiver, i.e., $\hat{B}_{\mathbf{r}}=\alpha$ where α is a scalar constant which corresponds to device sensitivity. We can show that the average intensity of $\Psi_g(\mathbf{r})$ is linked to $L(\mathbf{r})$ and $C(T)$ $[C(T)=2-b(T)]$ by the relation

$$I_g(\mathbf{r},\mathbf{r}_0=0;T)=1+C(T)\frac{[\hat{A}_{\mathbf{r}}\hat{B}_{-\mathbf{r}}L(\mathbf{r})]^2}{[\hat{A}_{\mathbf{r}}\hat{A}_{-\mathbf{r}}L(\mathbf{r})]_{\mathbf{r}=\mathbf{0}}[\hat{B}_{\mathbf{r}}\hat{B}_{-\mathbf{r}}L(\mathbf{r})]_{\mathbf{r}=\mathbf{0}}}. \tag{8}$$

If this expression is worked out for a dipolar emission and a monopolar reception, then we find again Eq. (6). A very interesting case is when $\hat{A}_{\mathbf{r}}=\hat{B}_{\mathbf{r}}$, i.e., when emitter and receiver are reciprocal. Intuitively, we could think, that as the reciprocity is restored, a monopolar cone should be found.

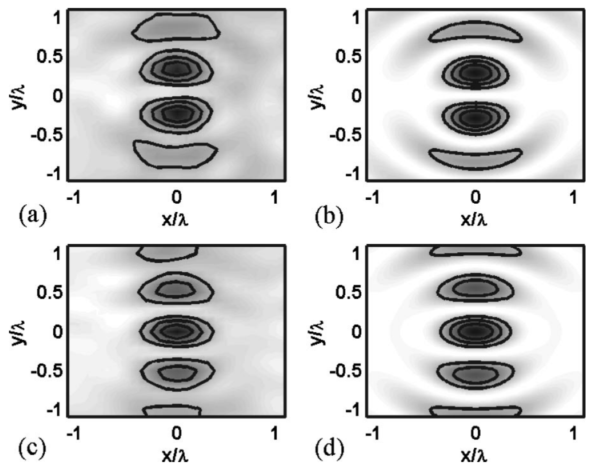

FIG. 5. Intensity pattern of the backscattered field for a dipolar source. (a) and (c): numerical simulations for a monopolar receiver and a reciprocal dipolar receiver, respectively. (b) and (d): predictions from Eq. (8).

But this is not the case. In fact, $[\hat{A}_{\mathbf{r}}\hat{A}_{-\mathbf{r}}L(\mathbf{r})]^2$ is not generally proportional to $L(\mathbf{r})$. For instance, we have been interested in dipolar reciprocal emitter and /or receiver. The emission axis is the same as the reception one. The theoretical pattern is represented on Fig. 5(d). We have checked this pattern with a numerical simulation similar to Weaver's one [9] [Fig. 5(c)]. We have also plotted patterns when the receiver is monopolar [Figs. 5(a) and 5(b)]: previous experimental results are found again. We clearly observe on Figs. 5(c) and 5(d) that for reciprocal devices, the monopolar and monopolar shape is not obtained. Nevertheless, the CBE is the same. Indeed, if $\mathbf{r}=\mathbf{0}$ and $\hat{A}_{\mathbf{r}}=\hat{B}_{\mathbf{r}}$ then the enhancement, $I_g(\mathbf{r}=0,\mathbf{r}_0=0;T)$, is always equal to $1+C(T)$. Therefore, the 'invariant' of the CBE seems to be only the maximum enhancement for full reciprocal experiments.

Emission and reception process imply the same kind of consequences on the CBE for systems where vectorial waves propagate. Typically, if a source generates the different field components with some weights that are different than those recorded by the receiver, then the CBE enhancement is lower than its maximum value due to the partially loss of reciprocity. B. A Van Tiggelen *et al.* have developed a related formalism in the context of multiple scattered seismic waves [6]. We have performed the same kind of analysis for reverberant elastic waves [12]. Indeed, we think that this vectorial effect, combined with dissipation effects, recently emphasized by Lobkis and Weaver [13] could explain the difference of amplitude observed between our monopolar and monopolar experiments with elastics waves and the scalar theory.

V. CONCLUSION

In this paper, we report experimental evidence of the role played by the source on the CBE for elastic waves. We have shown that using a dipolar source and a monopolar receiver, we obtain a''bicone'' instead of a simple cone that is well explained by describing dipolar as the superposition of two monopolar sources opposite in phases. This experiment thus implies a more general problematic: the influence of the double operation emission and reception on the CBE. Especially, we have shown that a full reciprocal experiment is not sufficient in order to recover the monopolar shape for the CBE.

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